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Physics 352 – Winter 2008

IMAGING, AND DIFFRACTION LIMITED RESOLUTION
(A SKETCH – JANUARY 30, 2008)

The Thin Lens

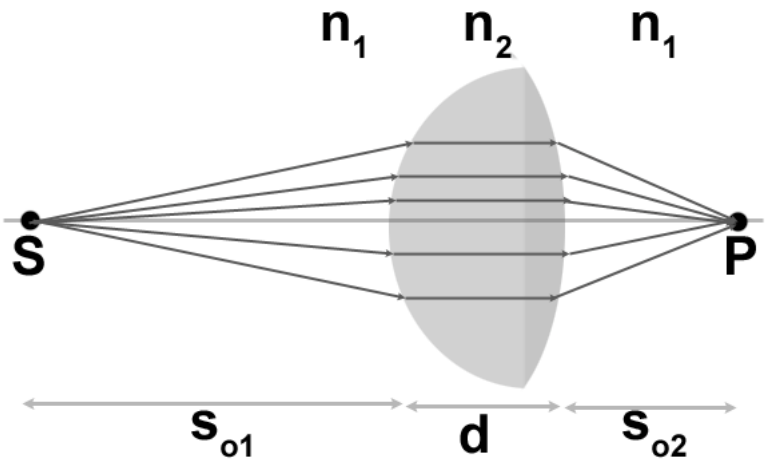


Figure 4.7. Focusing light with a thin lens (imagine d is small).

We showed:

$$\frac{n_1}{s_{o1}} + \frac{n_1}{s_{i2}} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a thin lens in air ($n_1 = 1$, $n_2 = n_{lens}$):

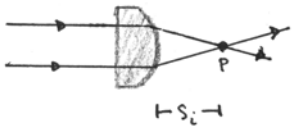
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_{lens} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The **focal length**, f, is given either by s_o or $s_i \to \infty$ (it doesn't matter which):

$\frac{1}{f} = (n_{lens} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. We can then write the thin lens equation as

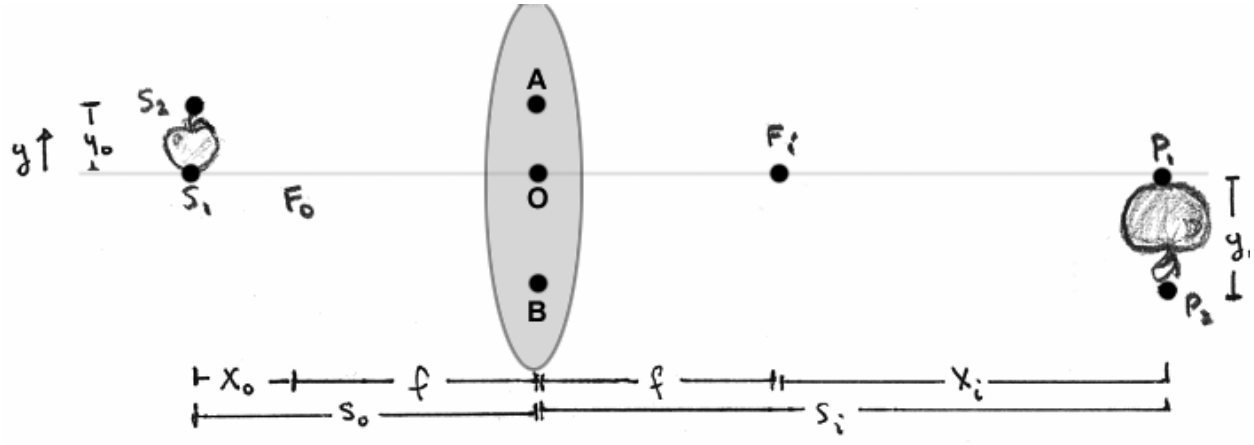
$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}$$, also known as the **Gaussian Lens Formula**. This is one the most important relations for the design of optical systems.

For example: Consider parallel rays incident on a glass ($n = 1.5$), $R_1 = \infty$, $R_2 = -50$ mm plano-convex lens. (See the figure, right, and note the relation between the shape of the lens surfaces and the signs of the R's.) Where will these rays be focused to? *Answer:* $\frac{1}{f} = (1.5 - 1)\frac{1}{50mm}$, so f = 100 mm, s_i = 100 mm.

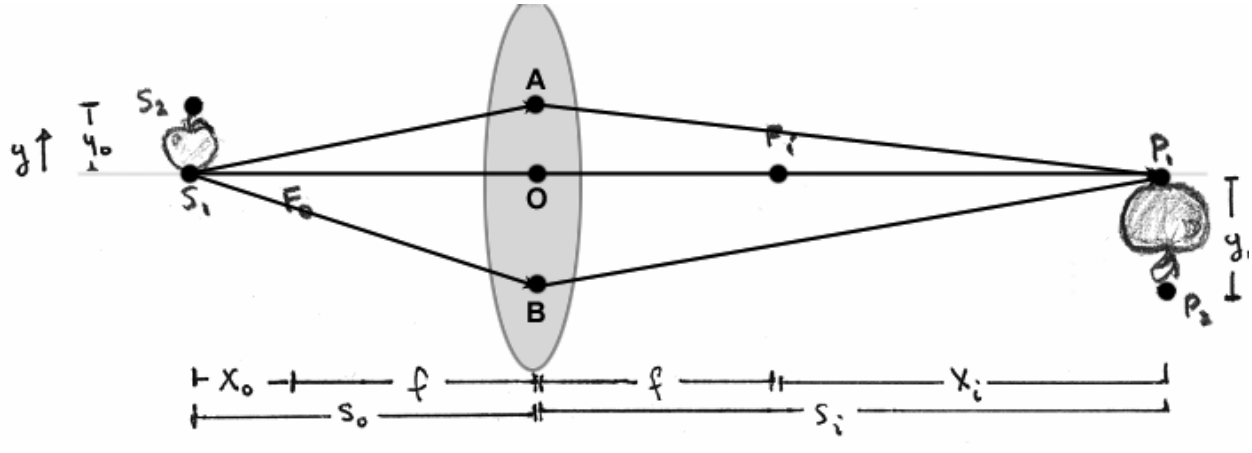


Imaging

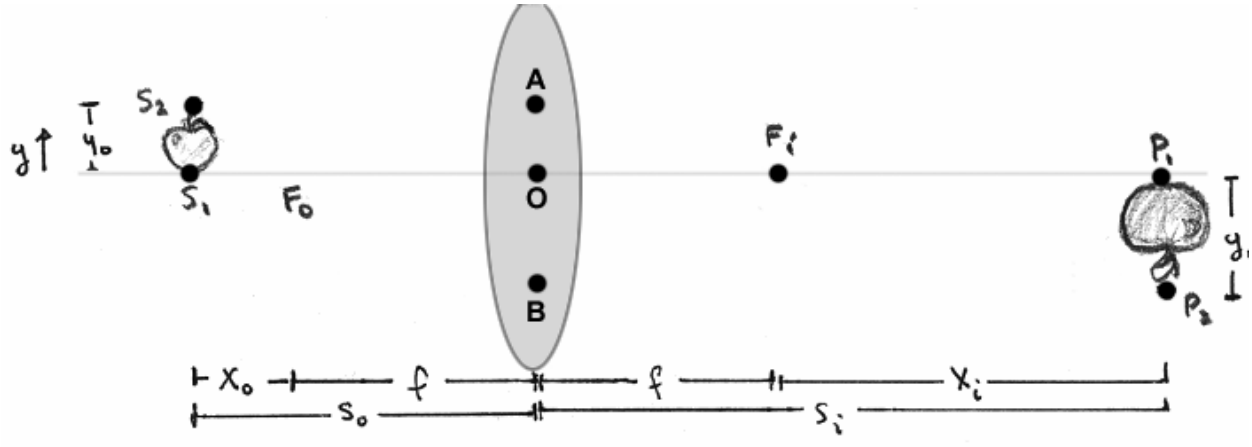
Consider a thin lens.



The distances s_o and s_i are related by the thin lens equations. We know that rays from the point S1 are focused to P1:

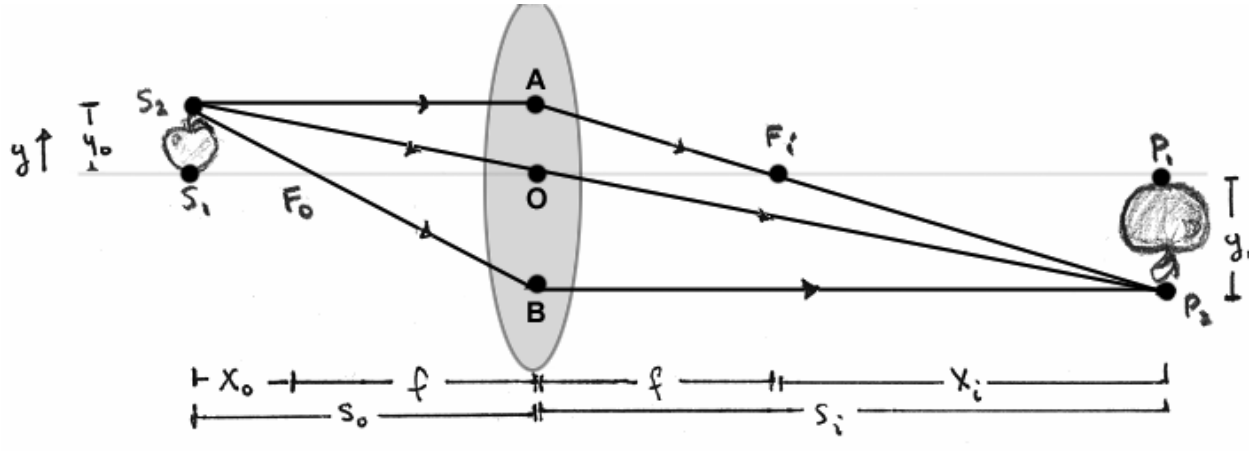


What about **off-axis rays**, for example from S2?



Ask: How is the ray S2-A bent? Do we know where it goes? (S2 and A are at the same y position.)

Answer: S2-A is parallel to the optical axis, and so is refracted to the focal distance, f.



Similarly, S2-B (the line chosen to intersect the focal point on the left) is refracted to be parallel to the optical axis on the right.

All these rays intersect at P2.

The y-position of P2 is the size of the image – the magnification that this produces is discussed in the notes; calculating it is a simple exercise in geometry.

If $s_o < f$, the rays do not converge at a real image point – this is just what we saw when discussing virtual images, that rays from an object placed closer than the focal distance do not converge to a point on the other side of the lens.

The key point we're concerned with is that **off-axis sources in the object plane are focused to off-axis points in the image plane.** The "image plane," like the "object plane" is perpendicular to the optical axis. Therefore, a lens can focus an image onto a screen!

This is how **eyes**, cameras, and all sorts of imaging devices work. The "image plane" in your eye is the retina.

Diffraction limited resolution

Why can't we see atoms?

We know that the angular resolution of an aperture of width a is $\approx \lambda / a$. Our lens above is also an aperture – **does its angular resolution set some sort of fundamental limit on what we can see?**

This questions seems puzzling – how can *angular* resolution determine a limitation of the *size* of what we can image? The dimensions aren't even the same, so it's not obvious how we might connect the two. Let's think...

Let's note the following:

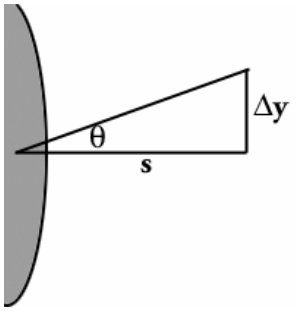
- The distance between the objects and the lens must be at least *f*, the lens' focal length (otherwise, as noted above, the lens can't construct a magnified image)
- We know the relation between *f* and the lens' radius of curvature *R*.
- We know that *R* can't be smaller than *a* (*Why?* – try drawing a spherical lens with $R < a$).

Throughout our analysis, we'll consider small angles, and not worry about "factors of 2," etc.

We'll consider a plano-convex lens, so that we can write $f = \frac{n_1}{n_2 - n_1} R$ -- i.e. just bothering with one radius of curvature. (This simplification won't affect our general result.)

Resolution.

Consider two objects separated in position by Δy at a distance s from the lens (see figure).



- For these objects to be resolvable, we need $\theta > \frac{\lambda}{a}$, where $\theta \approx \frac{\Delta y}{s}$.
- Therefore, we need $\Delta y > \frac{s\lambda}{a}$.
- Since $s > f$, $f = \frac{n_1}{n_2 - n_1} R$, and $R > a$, we can write $s > \frac{n_1}{n_2 - n_1} a$.
- Combining the two inequalities: $\Delta y > \frac{n_1}{n_2 - n_1} \lambda$.
- The numerical factor $\frac{n_1}{n_2 - n_1} \approx 1$ (though we can make it a bit smaller by designing a high-index lens).

Therefore, our minimum resolvable spatial separation is $\boxed{\Delta y_{min} \approx \lambda}$. **We can't resolve objects smaller than (approximately) the wavelength of light.** This "diffraction limit" is why

we can see using visible light macroscopic objects, or cells in a microscope (a few μm), but **not** atoms and molecules.

A more "quantitative" criterion that is often used is $\Delta y_{min} \approx \frac{\lambda}{2n_{lens}}$.

"Breaking" the diffraction limit

Can microscopy transcend the above "fundamental" limit to its spatial resolution?

In the past ten years or so, scientists have come up with clever schemes for "sub-diffraction" microscopy. Let's explore some of these...